International Journal of Waste Management and Technology
Vol. 1, No. 1, April 2013, PP: 14 - 22
Available online www.ijwmt.com

Research article

MATHEMATICAL MODEL TO PREDICT THE VELOCITY OF SOLUTE TRANSPORT IN SILTY AND FINE SAND FORMATION IN UNCONFINED BED IN PORT HARCOURT METROPOLIS,NIGER DELTA OF NIGERIA

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Abstract

Mathematical model to predict the velocity of solute transport in silty and fine sand formation in unconfined bed has been developed**,** the model were derived through the formulated governing equation, mathematical equation were developed considering the variables that influence the system, the model were derived to determine the velocity of solute with respect to period of migration and distance travelled to ground water aquifers, the rate of fast migration to ground water aquifers are through the formation characteristics such as porosity and permeability, these two parameters were the variables that played major roles in fast migration of solute to ground water aquifers, these two parameters determine the rate of velocity of fluid in soil, since solute are in fluid, the rate of fluid flow are determined through these stated conditions, the developed model will definitely monitor the rate velocity of solute in the study area.

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Keywords: mathematical model, velocity of solute transport in silty and fine and unconfined bed

1. INTRODUCTION

Among the many waterborne pathogens of humans, enteric viruses have the greatest potential to move deeply through the subsurface environment, penetrate aquitard, and reach confined aquifers. Enteric viruses are extremely small (27-75 nm), readily passing through sediment pores that would trap much larger pathogenic bacteria and protozoa. Viruses have been found in groundwater at depths of 67 m (Keswick and Gerba 1980; Robertson and Edberg 1997) and 52 m (Borchardt et al 2003) and lateral transport has been reported as far as 408 m in glacial till and 1600 m in fractured limestone (Keswick and Gerba 1980). Several recent studies have demonstrated widespread occurrence of viruses in domestic and municipal wells in the United States (Abbaszadegan et al 2003; Borchardt et al 2003; Fout et al 2003; Borchardt et al 2004 Borchardt et al 2007a Borchardt et al 2007b), and approximately half of waterborne disease outbreaks attributable to groundwater consumption in the United States have a viral etiology (National Primary Drinking Water Regulations, 2006 Kenneth, 2010).

The US Environmental Protection Agency has listed several viruses on its drinking water Contaminant Candidate List, emphasizing that waterborne viruses are a research priority (http://www.epa.gov/safewater/ccl/index.html). Although the vulnerability of groundwater to virus contamination is now recognized, the occurrence of viruses in confined aquifers has rarely been explicitly investigated. In the most comprehensive groundwater-virus study to date, Abbaszadegan and others (2003) sampled 448 groundwater sites in 35 states and found 141 sites (31.5%) were positive for at least one virus type.

Knowledge about the local hydrogeologic system and virus survival time makes some of these conceptual models more probable than others. The only environmental source of human enteric viruses is human fecal waste, and within the city limits of Madison human fecal waste is presumably only present in sanitary sewers. From this presumed point of entry, viruses must travel downward over 200 feet though the upper sandstone aquifer, an additional 10 to 30 feet downward through the Eau Claire aquitard to reach the top of the Mount Simon aquifer. Once in the Mt Simon aquifer the viruses must move laterally some unknown distance to the production wells. Based on such a travel path, pathway seems very unlikely because travel times would likely be far longer than the six months to two years these viruses can survive in the environment (Yates et al., 1985, John and Rose 2005, Schijven et al., 2006). Transport pathways 2 and 3, through breaches in the aquitard or through fracture pathways, are more probable, but one must still account for the long travel distance through the upper sandstone aquifer above the aquitard. Pathway 4, transport down the annulus of the well itself through deteriorated or poorly installed grout or through breaches in the well casing, seems the most likely mechanism for virus transport. This pathway could produce rapid downward movement of water with delivery directly to the well bore. During the previous virus study in Madison (Borchardt et al., 2007a) we collected limited samples for analysis of environmental isotopes. Tritium, deuterium, and oxygen- 18 have long been used in hydrogeologic studies to help distinguish groundwater age and source areas (Clark and Fritz, 1997). Previous tritium data suggested that Madison wells 5 and 24 produce relatively “old” groundwater (little or no tritium content), while well 7 produces “younger” water (tritium near the levels in modern precipitation). We hoped that oxygen-18/deuterium data would be useful in confirming or discarding flow paths

In general, however disease outbreaks related to microbiological contamination of water supplies tend to be more mundane and the source of illness may often remain indentified or unreported, particularly when related to groundwater supplies (National Small Flows Clearing House (USA), 1996). Nonetheless, the United States centre for disease control registered 318 waterborne disease outbreaks associated with groundwater systems between 1971 and 1996 in the US (Macler and Merkle, 2000). Similarly, the United States Environmental protection Agency

(USEPA) Science Advisory Board concluded that microbiological contaminants (bacteria, viruses and protozoa) were likely the greatest remaining health risk management challenge for drinking water suppliers. This later information has prompted the USEPA to propose the

Groundwater Rules which requires consumers of groundwater from supplies with at least 15 connections, or serving more than 25 people at least 60 days per year, to be protected against bacterial and viral contamination. The Groundwater Rule recognizes that the capacity for aquifers to remove microbiological contaminants from groundwater may not be adequate to protect public health, and requires groundwater supplies to achieve a 99.99%inactivation (loss of virulence) or removal of viruses, if groundwater is to besupplied without treatment. Information concerning microbiological contamination of groundwater inEurope is less widely available than in North America. This does not imply anabsence of microbiological contamination, rather a lack of widely available dataand an absence of coherent systematic monitoring strategies. Indeed, datapresented by Powell et al (2003) demonstrated that groundwatercontamination by faecal bacteria and viruses was widespread in the NottinghamAquifer, UK. Such craniological contamination may not pose a problem topublic health provided the water is not used for human consumption, or it istreated beforehand. However, groundwater is the main source of drinking waterin many European countries (European Commission, 1996). Furthermore, in many areas groundwater is not treated prior to entry into distribution systems,under the assumption that the water is pathogen-free. Clearly, the entry ofpathogenic micro-organisms into non-treating systems could pose a substantialthreat to public health. This issue is of particular concern with respect to aquifersclassified as sensitive to microbial contamination. Karst, fractured bedrock andgravel aquifers fall into this category of sensitive aquifers (USEPA, 1996).

2. THEORITICAL BACKGROUND

Velocity of solute transport in port Harcourt cannot over be emphasized, the rate of soil from the organic soil has resulted to a lot of water pollution in the study area, thousand of peopleare suffering from water related diseases in port Harcourt, the cause of fast migration of this pollution through fluid were not thoroughly investigated, and the settlers in the study area are suffering because could know the source of there ill health, death of thousand are caused by water related diseases in the country at large, this ugly siege cannot be over emphasized, because water is the life of man, the major source of water in the study area is ground water, solute migration from the soil to contaminate aquiferious zone is a serious threat to humane in the study area. Fast migration of this pollutant are caused by some certain factors and the factors if not known will continue to cause more harm to ground water, the soil have a lots of characteristics, this formation characteristics are developed by so many factors, the cause of fast migration of solute are the stratification of the formation, this condition are determined by degree of geological setting in the study area, to monitor the rate of solute fast migration, the rate fluid flow in soil should be the subject of concern. The rate of fluid are determined through the micropole of the soil, through porosity and permeability of the formation. This are the influence of fluid flow in soil formation, other conditions include the structural stratification of the entire formation through the disintegration of the sediment, this may on the process develop some fracture and also generate in some condition heterogeneous stratification, this variables developed variation of fluid flow in soil. The measurement of fluid flow within a short time are called velocity, this can be in solute or in ordinary fluid. The focus of this study is to monitor the velocity of solute in silty and fine sand formation in unconfined bed, the study through the investigation of hydrogeological studies were confound to deposit unconfined bed. Predominated all over the study area,this condition explains significant effect of velocity solute, in unconfined bed to ground water aquifers. For thorough investigation of the rate of velocity of solute, mathematical model were develop, this model were formulated through the variables that influence the velocity of solute in the formation. The governing equations are stated below.

3. GOVERNING EQUATION

$$Sop\frac{\partial^2 p}{\partial t^2} + \left[\varepsilon w\frac{\partial p}{\partial t}\right] w\frac{\partial p}{\partial t} - \frac{\partial p}{\partial x_1}\left[\frac{K_1 p}{\mu}\right]\left[\frac{\partial p}{\partial x_j} + pg\frac{\partial p}{\partial x_i}\right] = QP_z \quad \ldots\ldots\ldots\ldots \quad (1)$$

Taking Laplace transformation of (1)

$$\frac{\partial^2 p}{\partial t^2} = S^2 P_{(t)} - SP - P_{(0)} \quad \ldots\ldots \quad (2)$$

$$\frac{\partial p}{\partial t} = SP_{(t)} - P_{(t)} \quad \ldots\ldots \quad (3)$$

$$\frac{\partial p}{\partial t} = SP_{(t)} - P_{(t)} \quad \ldots\ldots \quad (4)$$

$$\frac{\partial p}{\partial x} = SP_{(x)} - P_{(x)} \quad \ldots\ldots \quad (5)$$

$$\frac{\partial p}{\partial x} = SP_{(x)} - P_{(x)} \quad \ldots\ldots \quad (6)$$

$$P = P_{(0)} \quad \ldots\ldots \quad (7)$$

The governing equation were transform into Laplace this to express the function to the condition were the variable will express there functions at different phase base on the influence of the velocity of solute in the transport process, subject to this transformation an expression were generated through the substitution from 21to 7 stated in equation 8
Submitting equation (2), (3), (4), (5), (6) and (7) into equation (1), yields

$$Sop\left[S^2P_{(t)}-SP_{(t)}-P_{(0)}\right]-\varepsilon w\left[SP_{(t)}-P_{(t)}-wSP_{(t)}-P_{(x)}\right]-\left[\frac{Kp}{\mu}\right]-\left[SP_{(x)}+Pg\frac{\partial p}{\partial x_j}+pgP_{(x)}\right]=QP_z \quad \ldots (8)$$

$$SoP_{(x)}-SoSP^1{}_{(t)}-P_{(0)}\ -\varepsilon w\,SP_{(0)}\ \varepsilon wP_{(0)}-wSP_{(t)}-P_{(0)}-\left[\frac{Kp}{\mu}\right]-\left[SP_{(0)}-P_{(x)}\ +Pg\ SP_{(x)}\right]=QP_z \quad \ldots.. \quad (9)$$

The expression from equation 8 were to relate the variable in the system with the transformation from equation 2 to 7 as express above, the relation with variable streamline the state of fluid flow in linear direction under the influence of formation characteristics in the system.

Considering the following boundary condition at

$$t=0, P^1{}_{(0)}=P_{(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots. \quad (10)$$

$$P_{(t)}\left[Sop\,S^2-Sop-\varepsilon w-w-\frac{Kp}{\mu}+Pg\right]=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots. \quad (11)$$

But considering the boundary condition

At $t>0,\ P^1{}_{(0)}=P_{(0)}=P_{(0)}$ ………………. (12)

$$P_{(x)}-Sop\,S_{(t)}-\varepsilon w\,S_{(t)}-w\,S_{(t)}-\frac{Kp}{\mu}\,S_{(x)}\,QPz\ =Sop\,P_0+SoPPo\ +\varepsilon wP_0\ +wP_0\ +\frac{Kp}{\mu}P_0 \quad \ldots (13)$$

$$\left[Sop-\varepsilon w-w-\frac{Kp}{\mu}-QPz\right]P_{(t)}=\left[Sops+Sop\ +\varepsilon w+w+\frac{Kp}{\mu}\right]P_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Boundary values were expressed subject to the variables in the system; these were integrated to monitor the limit of the velocity of transport in such stratification that influences fast migration of solute to the aquiferous zones.

This expression implies that the fluid flows are measured by the velocity, the variables in the system expressed the time of migration, based on the deposition of the formation in the study area. Equations (13) and (14) that the boundary values were expressed and integrated generated how the functions that influence the velocity of transport play their roles in the system.

$$\frac{P_{(t)}=Sop\ +\varepsilon w+w-\dfrac{Kp}{\mu}}{Sop-\varepsilon w-w-\dfrac{Kp}{\mu}-QPz}\,P_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

The variables in the system haven been integrated boundary values were coupled mathematically whereby the functions of various variables were expressed in detailed direction but cannot produce a result that will ascertain the measurement of fluid through velocity. To develop this expression, quadratic equation were introduced. Applying this expression we discretize various variable functions as they influence the velocity of transport either on solute or at ordinary state. These applications are introduced to stabilize the system and streamline the functions of the variables that influence the velocity of flow in the study location.

Applying quadratic equation, we have

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (16)$$

Where $a = Sop\varepsilon w, \quad b = \dfrac{wKp}{\mu}, \quad c = QPz$

$$\frac{-\dfrac{WkP}{\mu} \pm \sqrt{\dfrac{WkP^2}{\mu} + 4Sop\varepsilon w\, QPz}}{2Sop\varepsilon w} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (17)$$

$$C_{(t)} = A\exp^{\frac{\left[-\frac{WkP}{\mu} + \sqrt{\frac{-WkP^2}{\mu} + 4Sop\varepsilon w\, QPz}\right]_t}{2Sop\varepsilon w}} - \exp^{\frac{\left[-\frac{WkP}{\mu} + \sqrt{\frac{-WkP^2}{\mu} + 4Sop\varepsilon w\, QPz}\right]_t}{2Sop\varepsilon w}} \qquad \dots\dots\dots\dots\dots\dots \quad (18)$$

Subjecting equation (18) to the following boundary condition and initial value condition.

$$x = 0 \; C_{(0)} = 0 \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20)$$

Boundary conditions were expressed on the application of quadratic expression; this is to monitor the velocity of solute with respect to change and distance under the influence of formation characteristics on transport process to ground water aquiferous zone. These expressions are in line with other boundary values that were applied above. Subject to this relation, the expressions that determine the velocity of solute at this phase were based on variation of formation characteristics of soil stratification in unconfined locations.

We have *B = -1 and A = 1* ……………………………… (21)

So that our particular solution, will be in this form

$$C_{(t)} = A\exp^{\frac{\left[-\frac{WkP}{\mu} + \left(\frac{-WkP^2}{\mu} - 4Sop\varepsilon w\, QPz\right)^{1/2}\right]_t}{2Sop\varepsilon w}} = \exp^{\frac{\left[-\frac{WkP}{\mu} + \left(4Sop\varepsilon w\, QPz\right)^{1/2}\right]}{2Sop\varepsilon w}} \qquad \dots\dots\dots\dots\dots\dots \quad (22)$$

But $e^x = e^{-x} = 2Sinx$

Therefore the expression of (22) can be written in this form

$$C_{(t)} = 2\; Sin\left[\frac{WkP}{\mu} + \left(\frac{WkP^2}{\mu} + 4Sop\varepsilon w QPz\right)\right]^{1/2} t \qquad \dots\dots\dots\dots\dots\dots \quad (23)$$

But if $t = \dfrac{x}{v}$

Therefore, the model can be expressed as:

$$C_{(x)} = 2\,Sin\left[\frac{WkP}{\mu} + \left(\frac{WkP^2}{\mu} + 4Sop\varepsilon w\,QPz\right)^{1/2}\right]\frac{x}{v} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (24)$$

But if $\frac{x}{v} = t$, we have

$$C_{(t)} = 2\ Sin\left[\frac{WkP}{\mu} + \left(\frac{WkP^2}{\mu} + 4Sop\varepsilon w\,QPz\right)^{1/2}\right]^{t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (25)$$

Considering equation (24) and (25) yield

$$C_{(x,t)} = 2\,Sin\left[\frac{WkP}{\mu} + \left(\frac{WkP^2}{\mu} + 4Sop\varepsilon w\,QPz\right)^{1/2}\right]x + 2\,Sin\left[\frac{WkP}{\mu} + 4Sop\varepsilon w\,QPz\right]^{1/2} \quad \ldots\ldots\ldots\ (26)$$

The expression in (26) is the final model equation that will monitor the velocity of solute that influences fast migration of contaminant to groundwater aquiferous zone. The derived mathematical equations were generated through the governing equation that will monitor the velocity of solute to groundwater aquifers. Fast migration of contaminants within a short period of time has been a serious concern to environmental experts. This is because a lot of ground water pollutant within a short period has been observed in deltaic environment. This has resulted to a lot of water-related diseases which investigation of the cause of these diseases were ignoredin the study location. The victims of this source of pollution were not able to know the source of their illness, groundwater are the major source of water for human utilization. Thousands of people in the study area got their water from public water supply and private boreholes. Subject to this relation, the rate of water related diseases cannot be overemphasized due to this ugly surge.

To investigate the source of this pollutant, hydrological studies carried out by hydro geologists were applied to determine the geological setting of study area. The information's from there studies were use to understand the aquiferious zone, since the geological formation of the study area were confirmed from the source, to monitor the velocity of transport development of mathematical mode were found to monitor the velocity of solute in unconfined bed the derived model expression were developed through the governing equation that were formulated, the variable consider generated the governing equation the equation derived generated the model that will monitor the velocity of solute in unconfined aquifers.

4. Conclusion

Velocity of solute in soil are determine by several factors, the rate velocity of fluid flow are determine on the soil structural deposition, these are influence by the degree of porosity of the soil formation, the study area are deltaic environment, this condition implies formation has a lots of environmental influence through climatic condition including the activities of man. The rates of industrial waste generate lots of pollution, but the rates of pollution migrating to ground water aquifer are the subject of concern in the study area. Fast migration are determined by the formation characteristics through the micropoles degree of depositions, this condition determine the rate of velocity of flow in strata to ground water bed deposit shallow aquifers, the formation were also confound to deposit homogenous soil formation, this condition implies that the migration will be in fast states base on this deposited influence in the study location, to monitor the rate of velocity of transport in the study area, mathematical model were found to be the absolute concept that determine the rate of solute transport in the study area. The model were derived through the governing equation developed to solve the problem, the governing equation were derived considering several conditions that influence the velocity of solute in deltaic environment, the derived mathematical model will monitor the velocity of solute in unconfined bed in the study area.

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